Contemporary strain rates in Fennoscandia from BIFROST GPS

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Baseline Inferences From Fennoscandian Rebound Observations, Sealevel and Tectonics

- *Hans-Georg Scherneck, Chalmers*
- *Rüdiger Haas, Chalmers*
- *Martin Lidberg, National Land Survey of Sweden*
- *Jan Johansson, Chalmers and SP Swedish National Research and Testing Institute*
- *Markku Poutanen, Hannu Koivula, Finnish Geodetic Institute*
- *Oddgeir Kristiansen, Norwegian Mapping Agency*
- *Glenn A. Milne, Univ. Durham, now Univ. Ottawa.*
- *James L. Davis, Harvard-Smithsonian Center for Astrophysics, Cambridge, Mass.*

Continuous GPS, daily solutions since Sep. 1993 using (16 …) 64 stations

Done or in progress:

Glacial Isostatic Adjustment (GIA)

Model inversions for earth rheology and ice history

(examining 3D displacement rates)

Observed absolute sea level change 1.2 ± 0.2 mm/yr ————–>

New:

Examine strain rates (”tectonics”)

feasible, owing to the increasing consistency of observations:

- time series length, stable observing conditions
- IGS improvements, maturing systems
- (scale parameter rates may be a problem, translations and rotations not)

Would like to have a tool for the interpretation of the deformation field, especially the GIA residual

- as **<u>general</u>** as possible
  (as an alternative to a specific model like GIA)
- emphasising the lithosphere
- emphasising (the not so well examined) horizontal motion
- enhancing anomalous regions (strain accumulation)

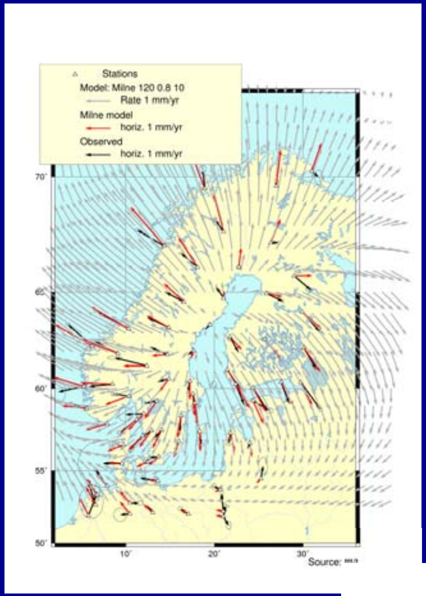
General: Leclerc (right)

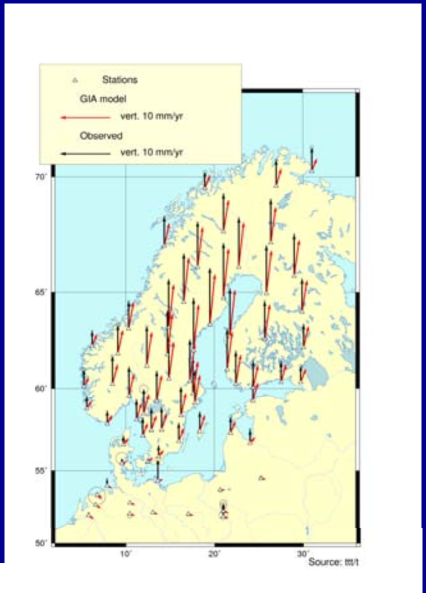
A: Strain rates
A-a: Using collocation (statistics)
A-b: Using a mechanical model

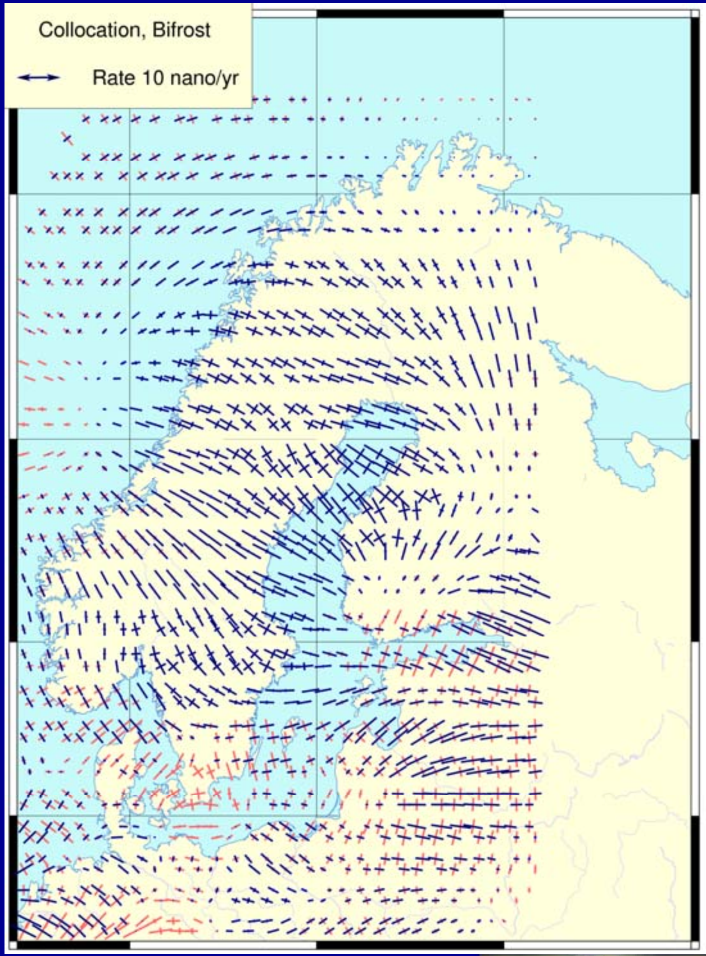
B: Something else (?)

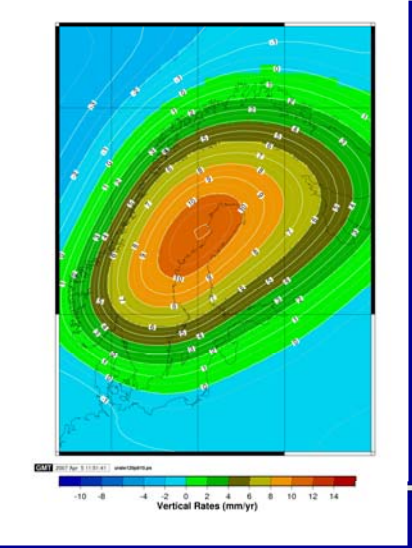
something else

Collocation









Thick-plate concept

Motivation:

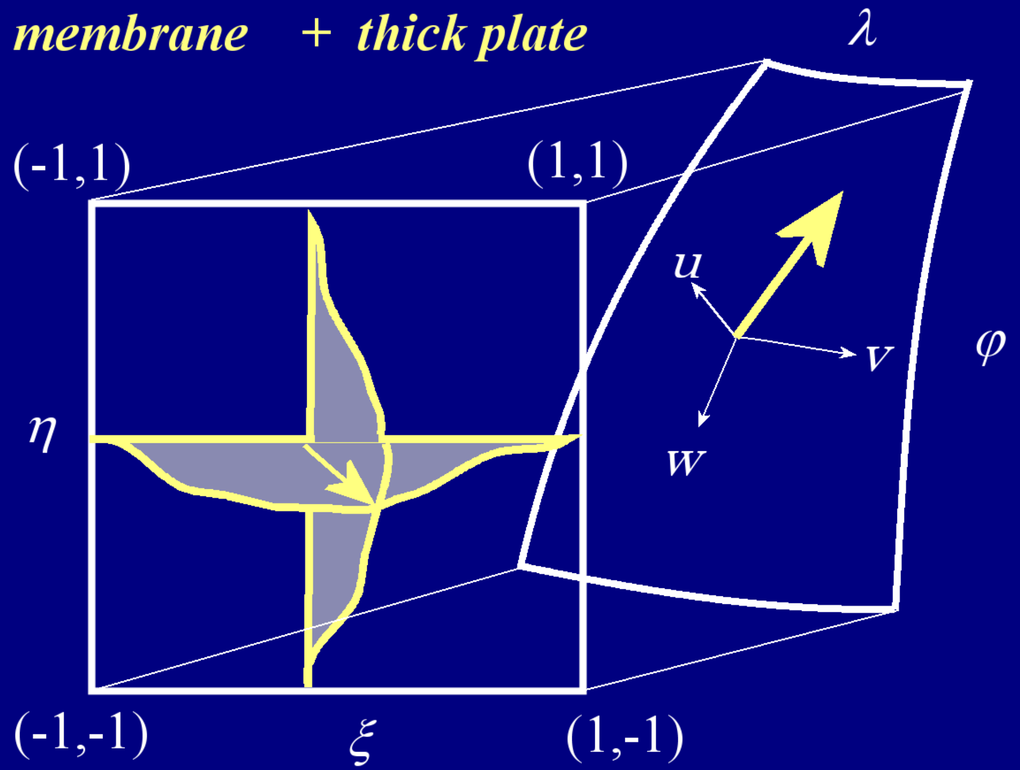
- Collocation treats strain as a stochastic parameter
- An elastic plate would reduce freedom to a physically consistent field
- A thin plate would not make use of the vertical velocity component as all stations are on a surface
- After all the lithosphere *is* thick, 200 km versus 2000 km diameter of rebound area
- Flexure is important if vertical velocity = 10 mm/yr at wavelengths < 1000 km

"Finite Element" [sic!]: one element, high polynomial order, free-slip boundary conditions

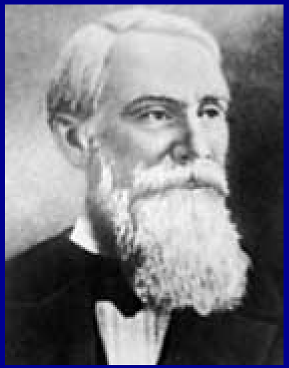
$$(u, v, w) = \sum_{i,j} (U_{ij}, V_{ij}, W_{ij})\, T_i(\xi)\, T_j(\eta)$$

$$\epsilon_{\phi\phi} = \frac{1}{R}\frac{\partial w}{\partial \phi} + \frac{u}{R} - \frac{\zeta}{R^2}\left[\frac{\partial^2 u}{\partial \phi^2} - \frac{\nu}{\sin^2 \phi}\frac{\partial^2 u}{\partial \lambda^2}\right] \quad etc.$$

membrane + thick plate



T = Chebychev Polynomials



$$\epsilon_{\phi\phi} = \frac{1}{R}\left(\frac{\partial v}{\partial \phi} + u\right) \tag{2}$$

$$\epsilon_{\lambda\lambda} = \frac{1}{R\sin\phi}\left(\frac{\partial w}{\partial \lambda} + \sin(\phi)u + \cos(\phi)v\right) \tag{3}$$

$$\epsilon_{\phi\lambda} = \frac{1}{2R\sin\phi}\left(\frac{\partial v}{\partial \lambda} + \sin\phi\frac{\partial w}{\partial \phi} - 2\cos(\phi)w\right) \tag{4}$$

$$\omega_z = \frac{1}{2R}\left(\frac{\partial w}{\partial \phi} - \frac{1}{\sin\phi}\frac{\partial v}{\partial \lambda} + \cot(\phi)w\right) \tag{5}$$

$$\epsilon_{\phi\phi}^{(\mathrm{TP})} = -\frac{z}{R^2}\left(\frac{\partial^2 u}{\partial\phi^2} - \nu\frac{1}{\sin^2\phi}\frac{\partial^2 u}{\partial\lambda^2}\right) \tag{8}$$

$$\epsilon_{\lambda\lambda}^{(\mathrm{TP})} = -\frac{z}{R^2}\left(\frac{1}{\sin^2\phi}\frac{\partial^2 u}{\partial\lambda^2} - \nu\frac{\partial^2 u}{\partial\phi^2}\right) \tag{9}$$

$$\epsilon_{\phi\lambda}^{(\mathrm{TP})} = -\frac{z}{R^2\sin\phi}\frac{\partial^2 u}{\partial\phi\partial\lambda} \tag{10}$$

Cost function

$$P = \frac{1}{3N_O}\sum_{Obs}(\mathbf{u}-\bar{\mathbf{u}})^\top \mathbf{\Sigma}^{-2}(\mathbf{u}-\bar{\mathbf{u}})$$

observation misfit

$$+\frac{\Omega}{N_E}\sum_{E-points}\left\{\int_{-D/2}^{D/2}\boldsymbol{\sigma} : \boldsymbol{\epsilon}\, dz\right.$$

plane stress and flexure work

$$\left. + g\rho_{eff}u^2\right\}$$

gravity work

$$= \frac{1}{3N_O}\chi^2 + \frac{\Omega}{N_E}\left(E_b + E_p + E_g\right)$$

Ω small: observation dominated

Ω big: strain energy dominated

Today: only observation dominated!

Uncertainty analysis of inferred strain rates

- Vary each Chebychev coefficient (say: *j*)
  such that the χ^2 of the observation misfit doubles
  (for 1 degree of freedom, this implies 85% confidence – N.B. we vary one coefficient at a time)

- Compute RMS:
  Root-mean of
  {deviation$_j$ of strain field}^{2} / number of coefficients of all *j*

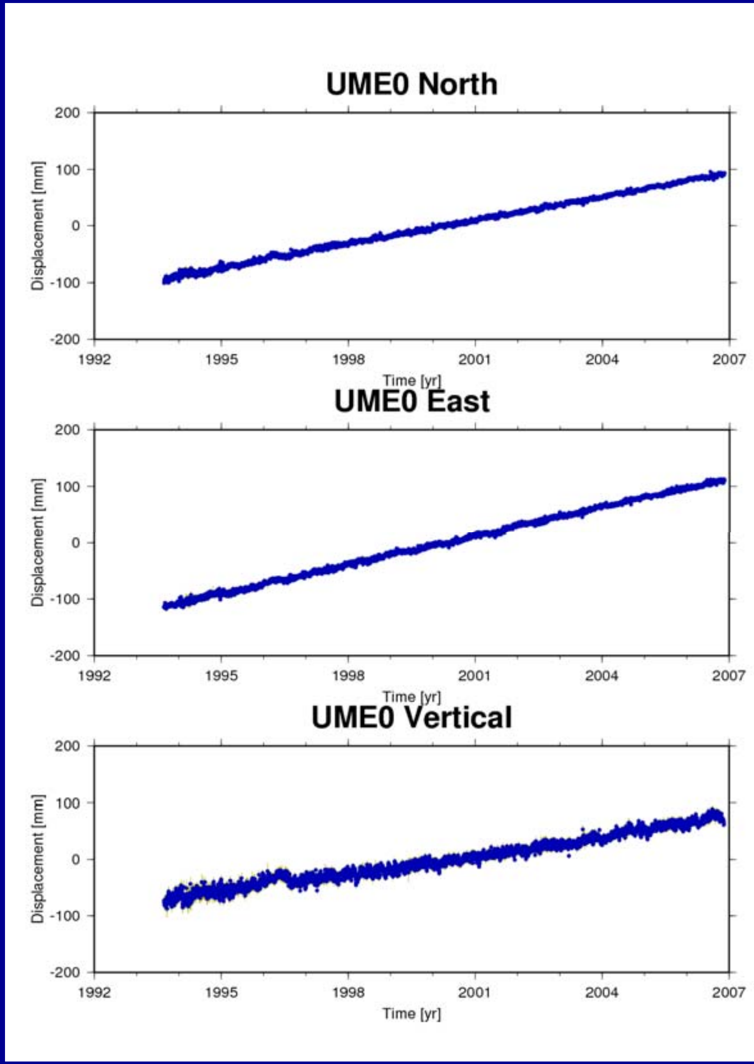
A typical normalised χ^2 of fit of observations is 10

Observations

- GPS 1996-2007
- daily solutions, 84 stations
- GAMIT
- ITRF2005

  + adjustment for vertical rates by local stabilisation

  (Lidberg, PhD-thesis May 2007)



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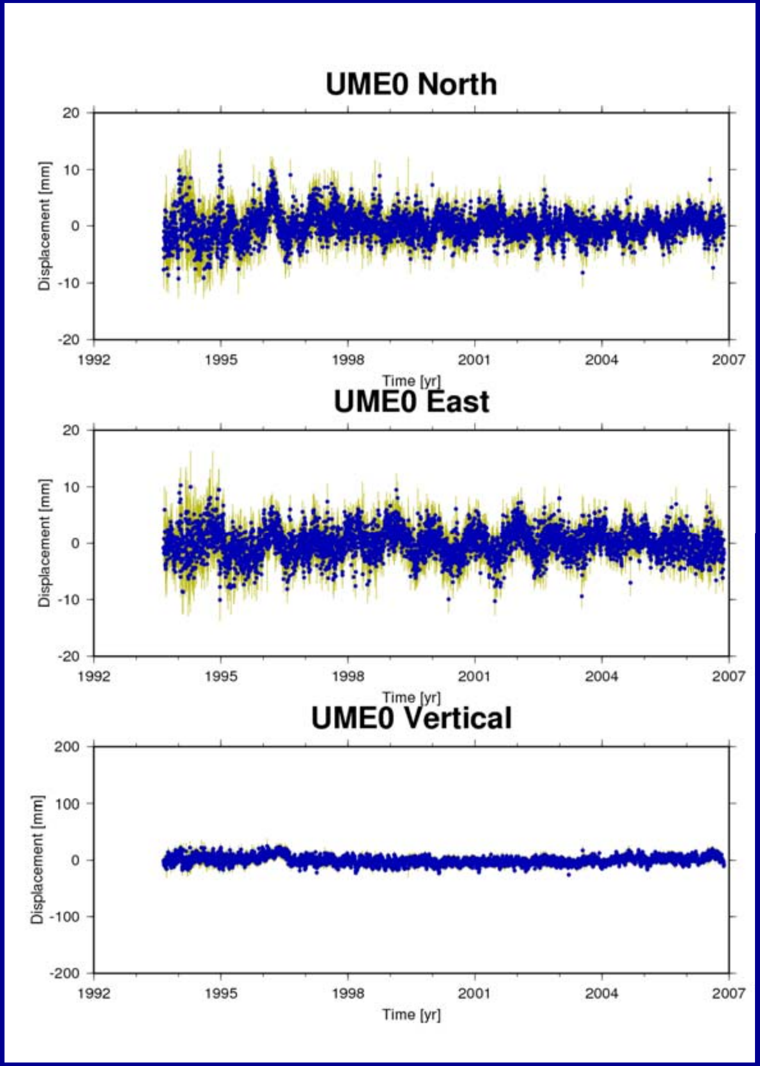
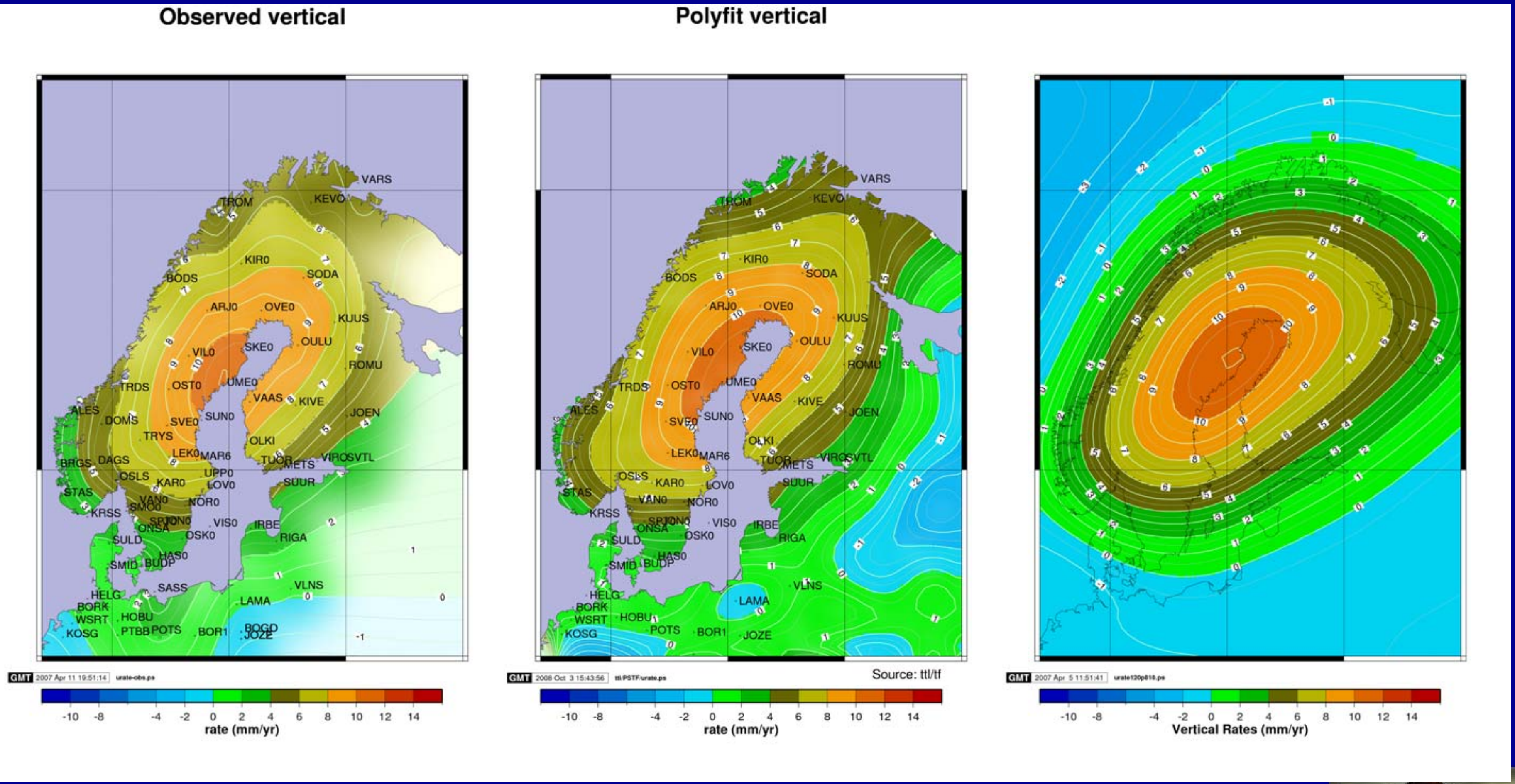
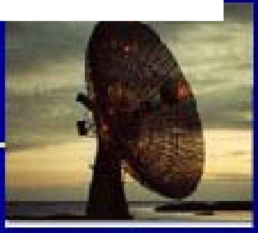


BIFROST project

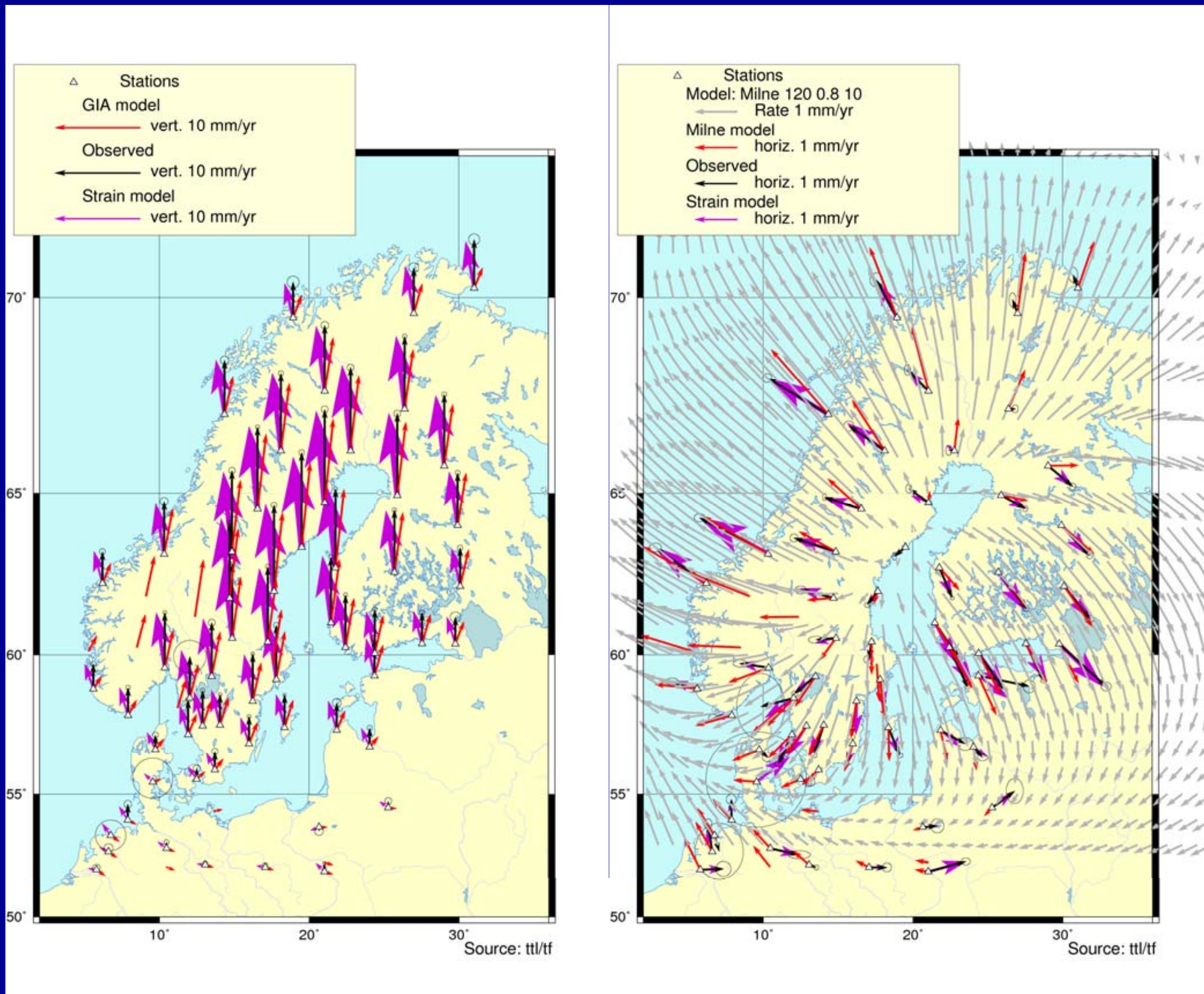
|--------------------- BIFROST – GPS -----------------------|

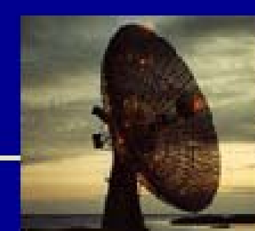
interpolated (raw) | thick-plate model | GIA model





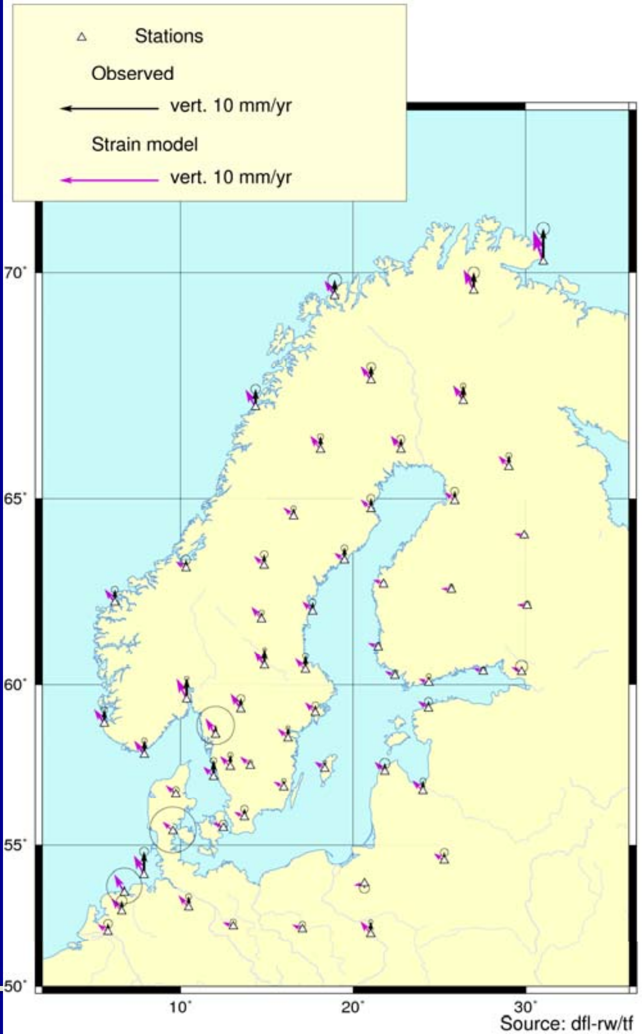
BIFROST project

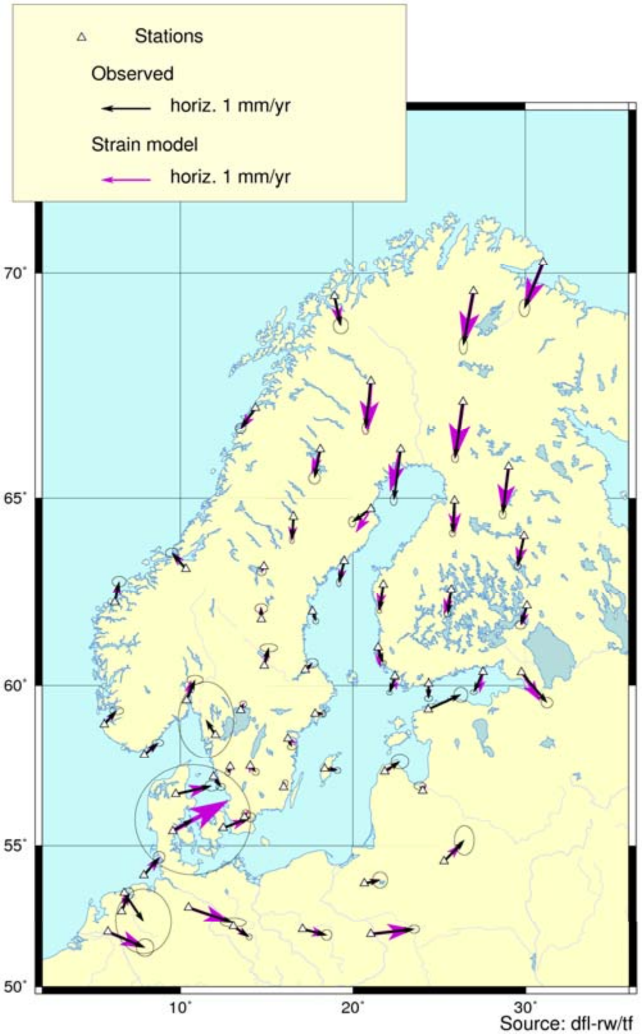




BIFROST project

"Observed" = GPS Observations – GIA model





Results of the strain analysis

Data sets:

- The GIA model on its own grid
- The thick plate model
  - GIA model
  - GPS
  - GPS-GIA residual

Products

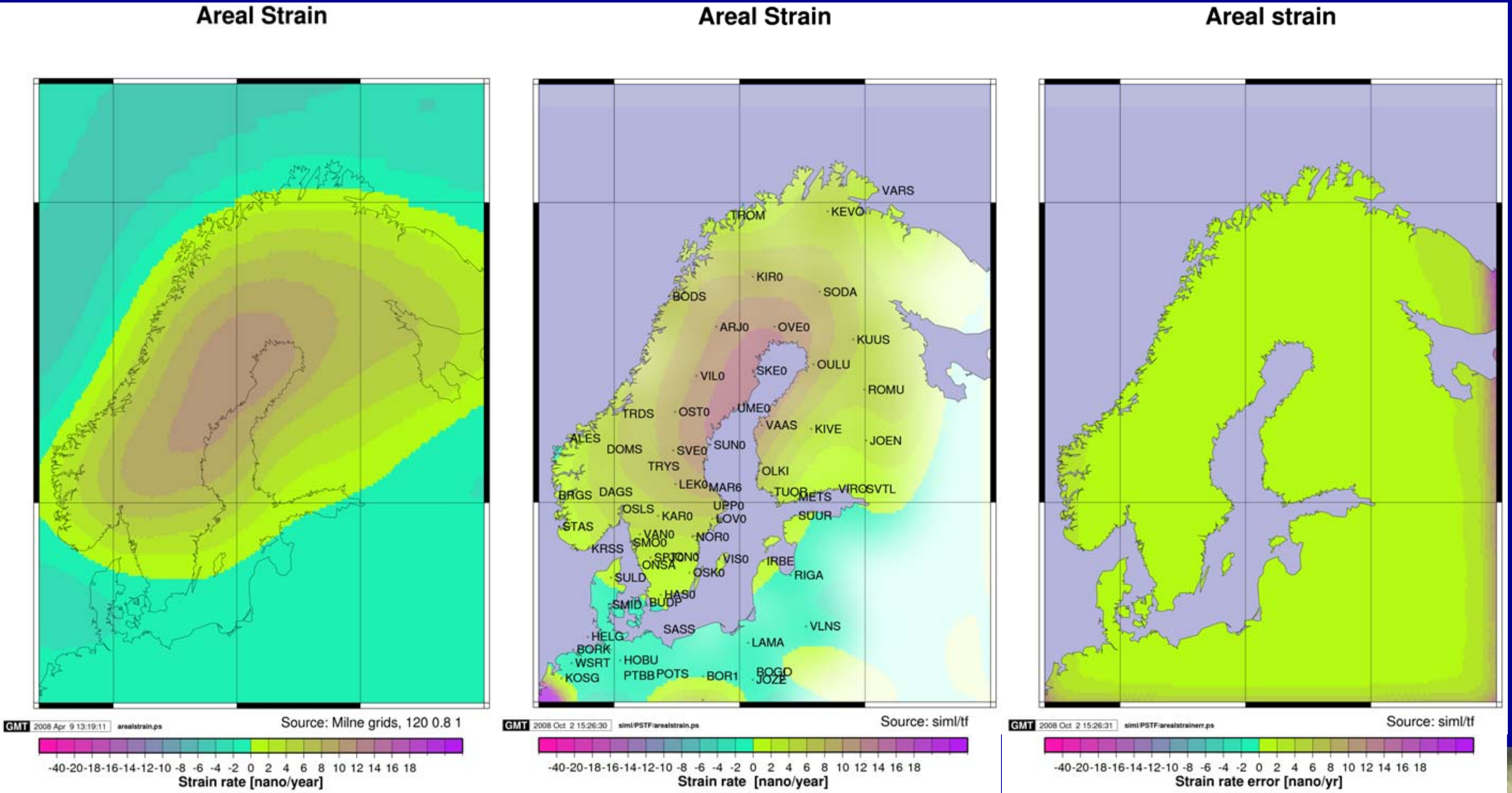
- Areal strain
- Shear strain
- Curl
- Uncertainties for each (3)
- Strain energy: Plane and bending, areal and shear (4)
- Observation misfit

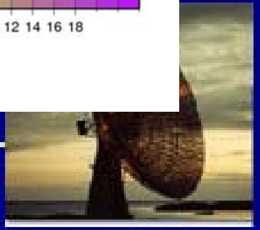
=> 33 + 3 = 36 maps to show!

First showing…

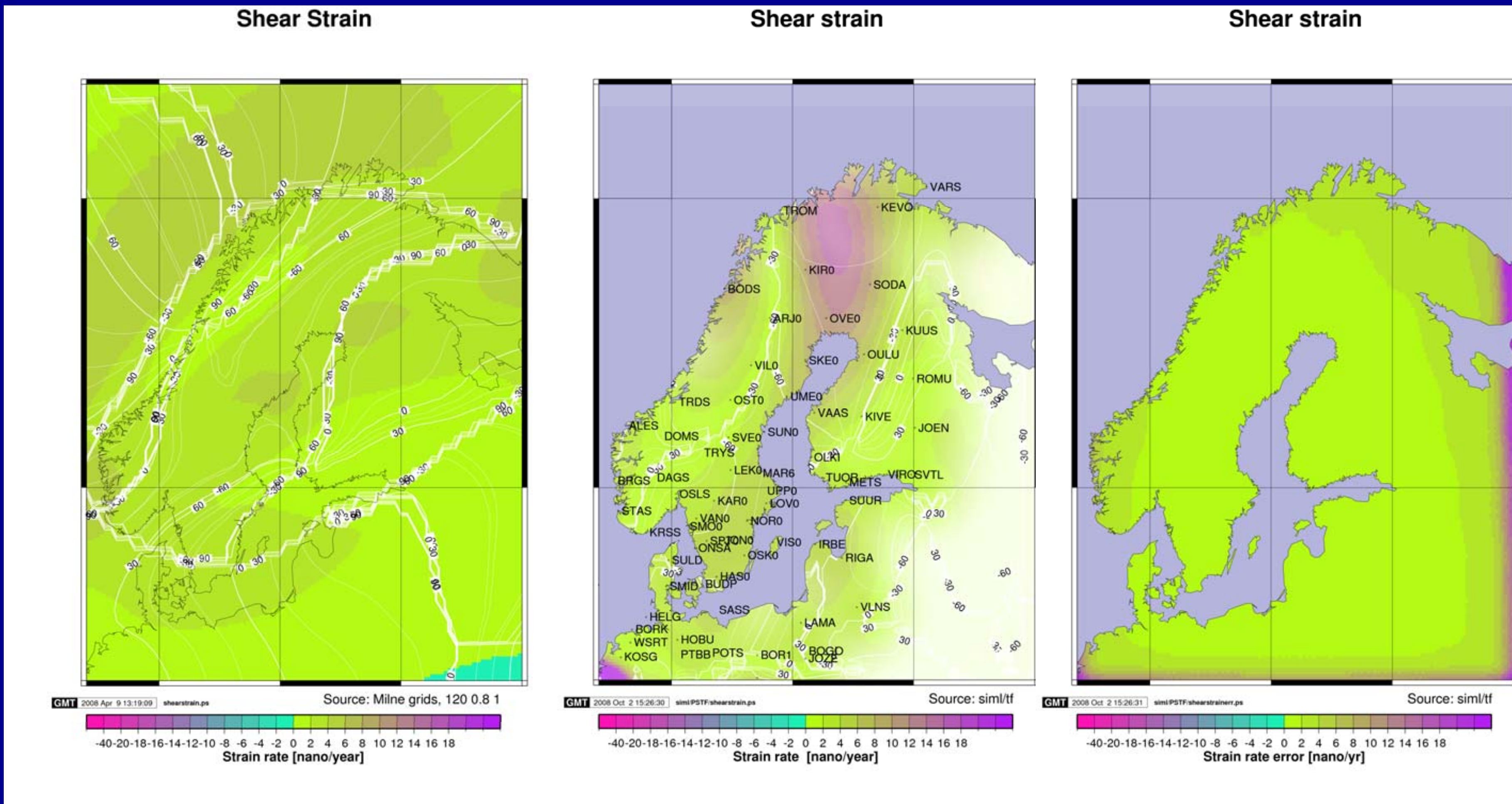
- GIA Model on a regular grid 1/12 x 1/12
- Thick-plate solution to GIA model
  - Represented at the BIFROST sites
  - Using the observation uncertainties
- Strain and curl Uncertainty

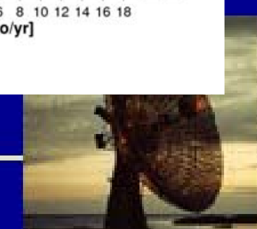
GIA model grid |--GIA model on BIFROST netw.--|





GIA model grid |--GIA model on BIFROST netw.--|

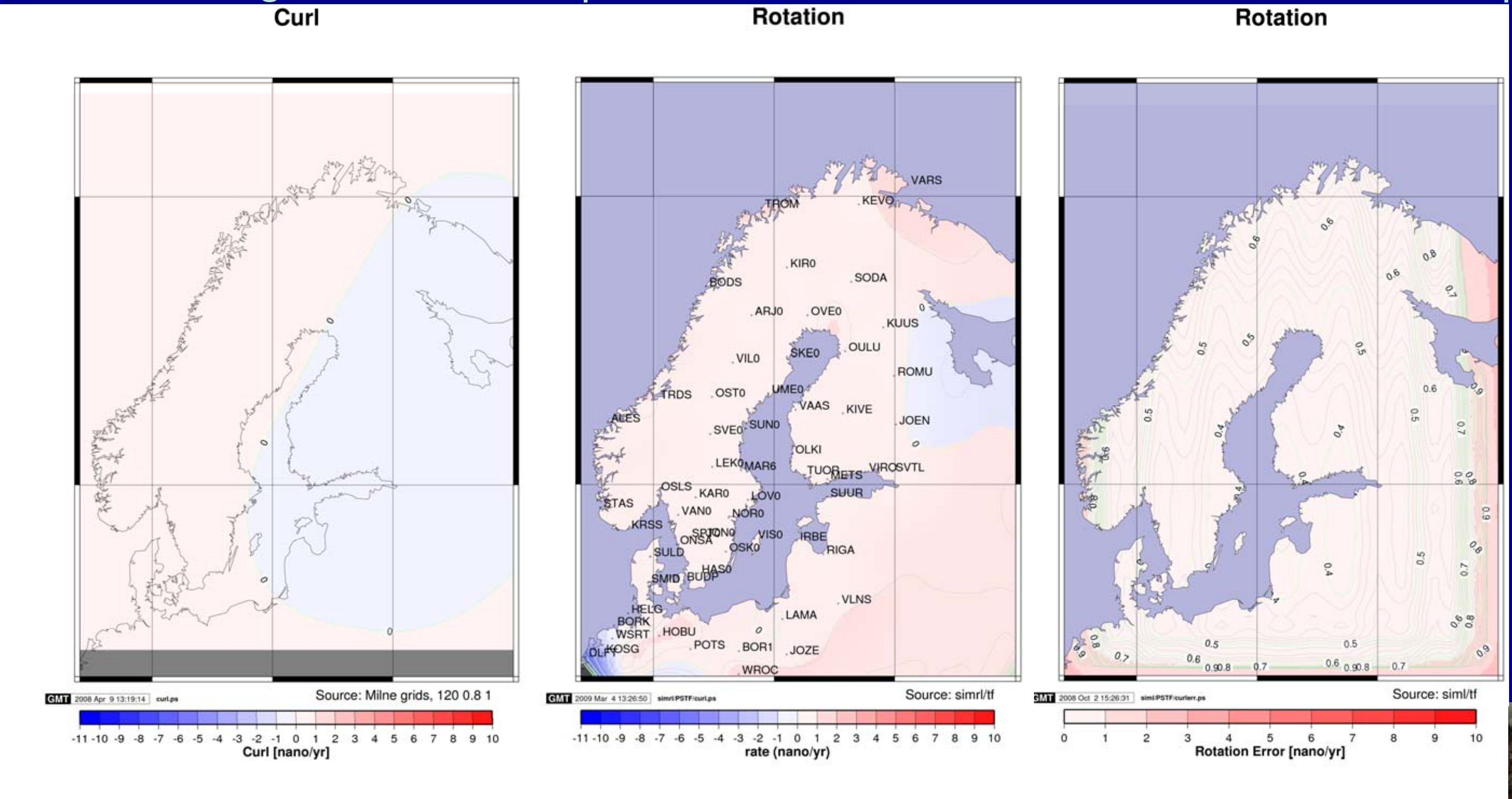


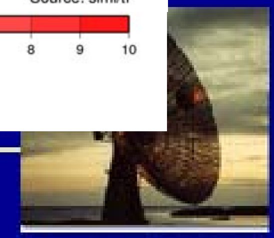


BIFROST project

Curl = Rotation. Theoretically, on a radially symmetric earth loaded by normal forces there is no curl.

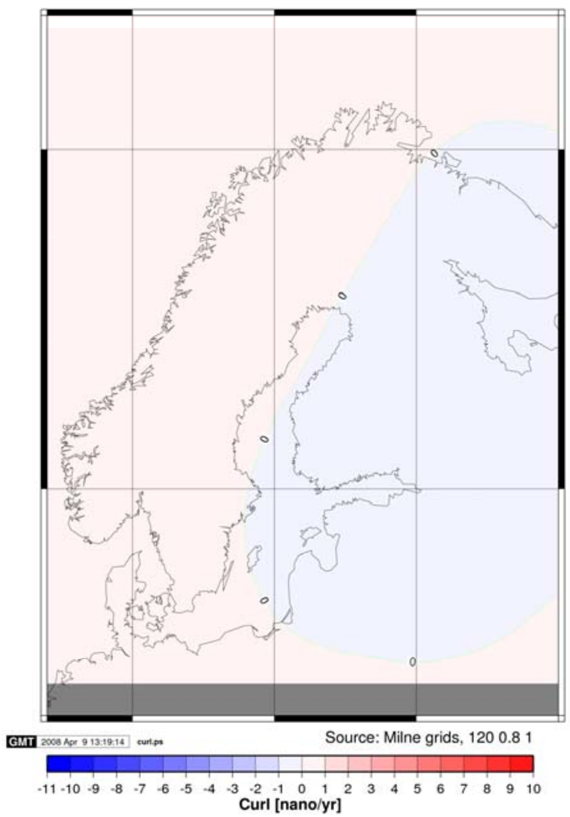
GIA model grid |---------- GIA model on BIFROST netw. ---------|



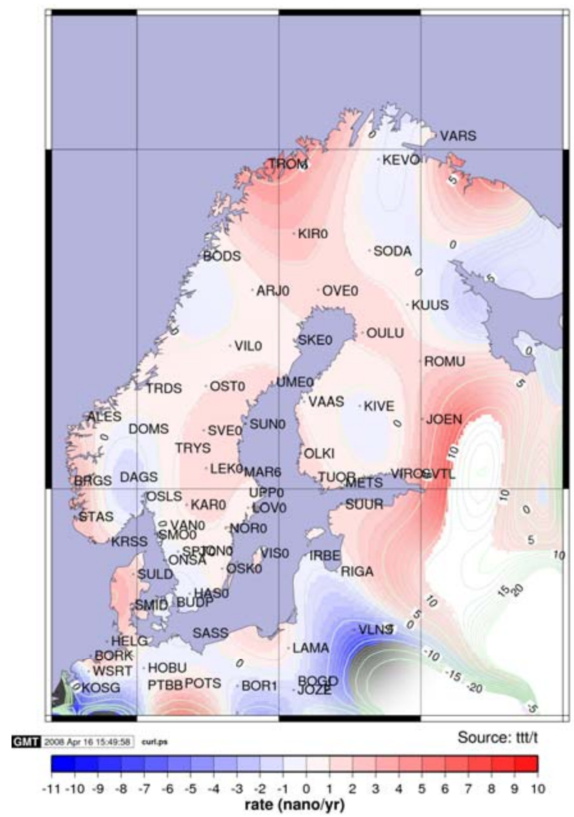


GIA model grid |--------------------- BIFROST GPS ----------------------|

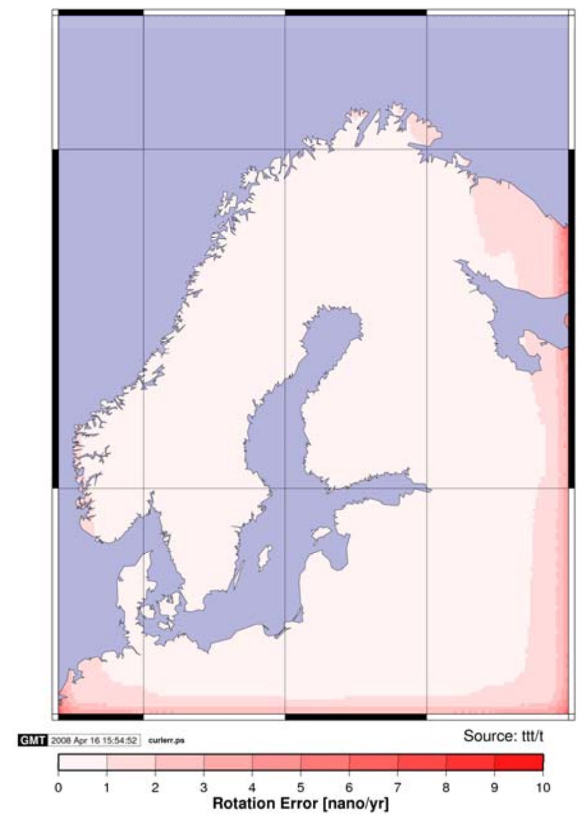
Curl

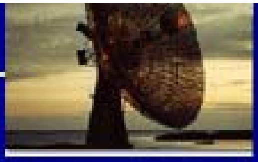


Rotation



Rotation





Nota bene

The curl does not vanish when the thick plate is fit to curl-free model data!

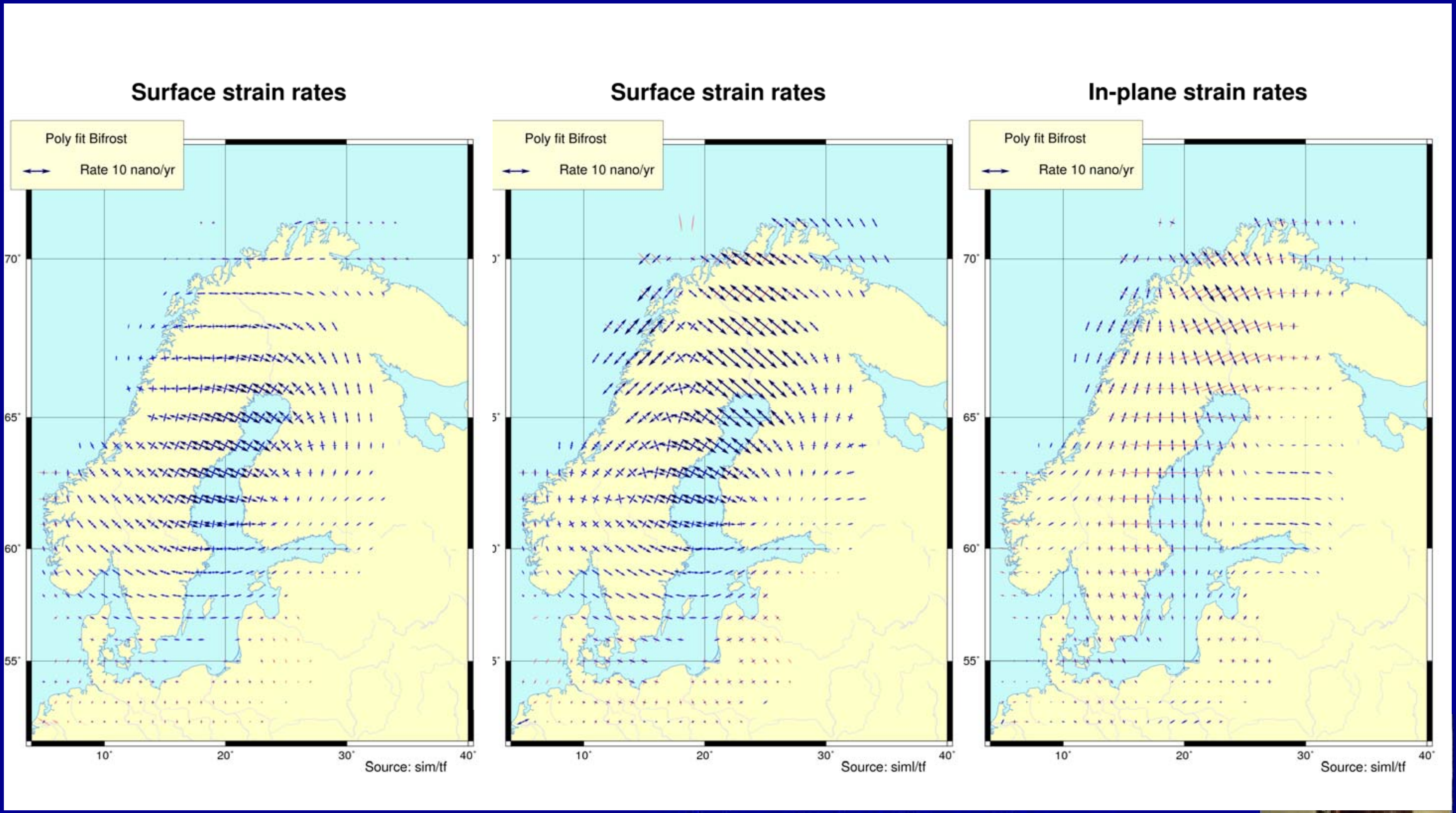
Similarly, shear strain is amplified

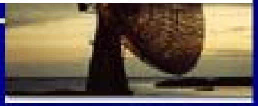
The curl would be one tectonic parameter, so there is no motivation to force it to zero or to issue an extra penalty.

The anomalies occur primarily where the station network i sparse

Input: GIA model data

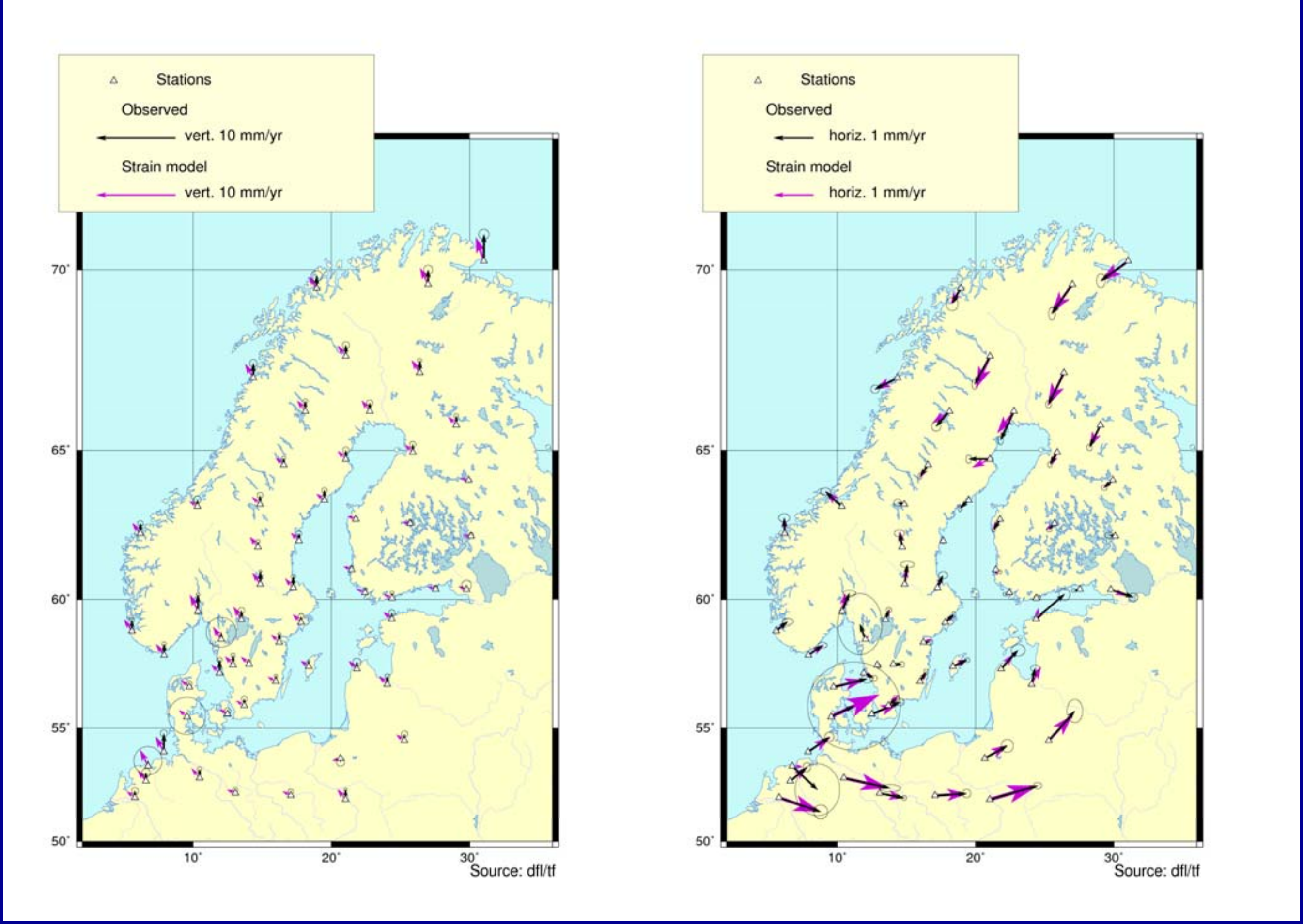
Thin plate |-------------------------- Thick plate --------------------------|





Next: Doing the analysis on the residual Obs - GIAmodel

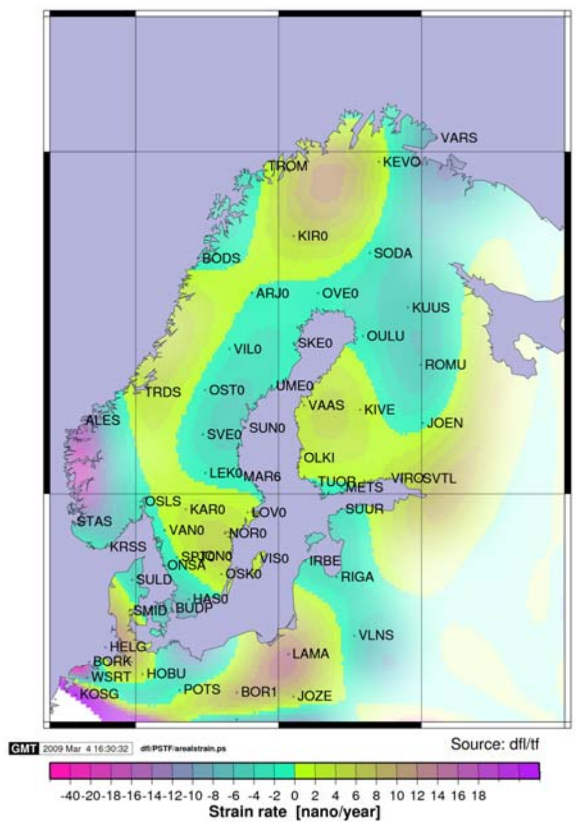
GIA + obs Strain m + obs



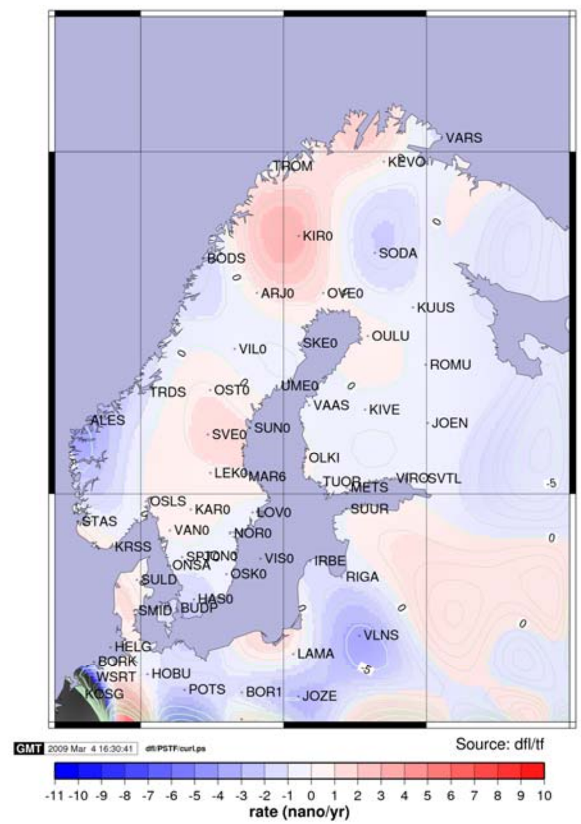
BIFROST project

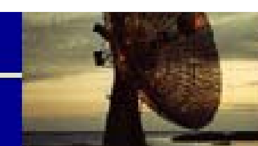
Surface strain rates

Areal Strain



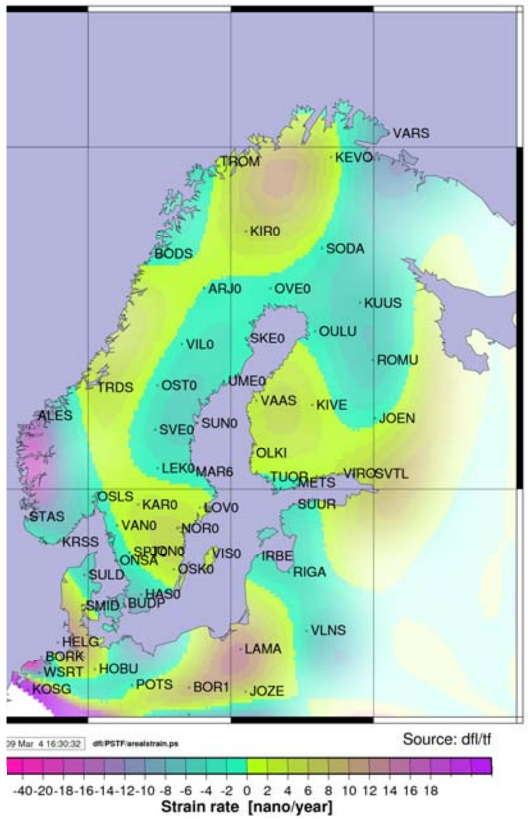
Rotation



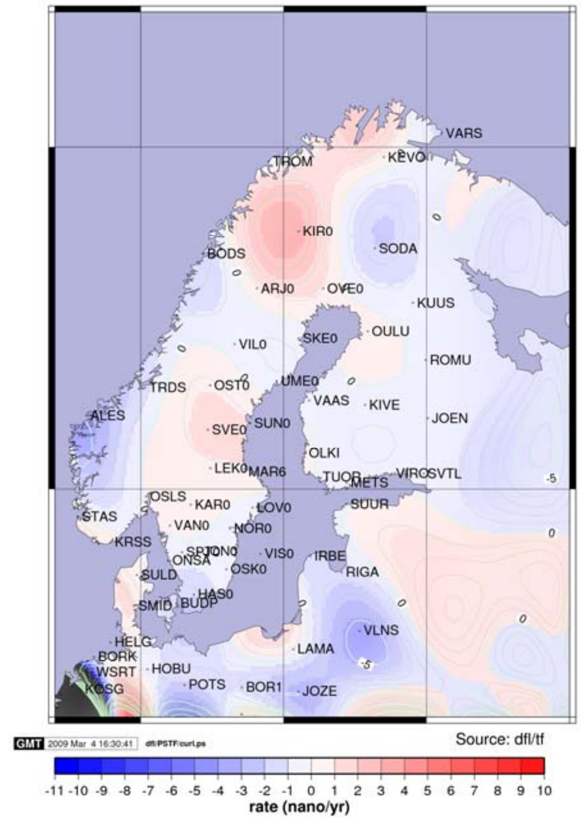


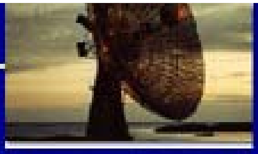
Shear strain

Areal Strain



Rotation

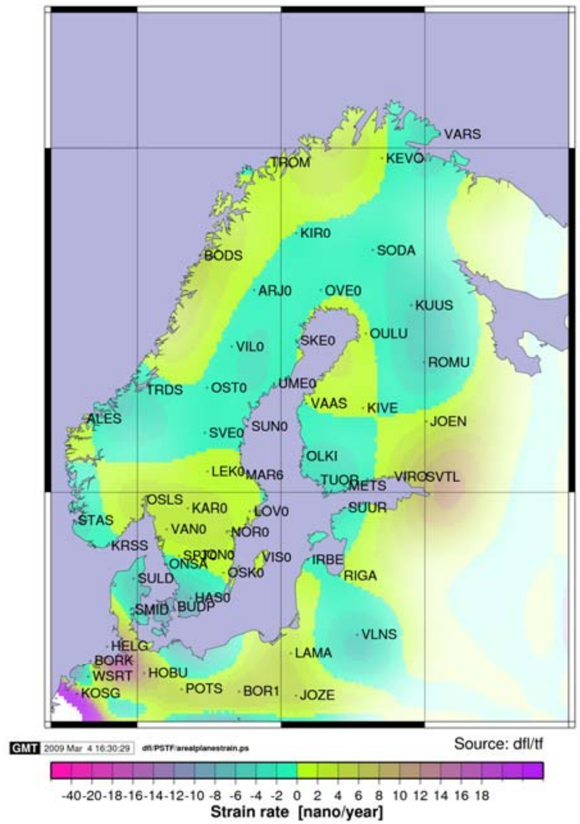




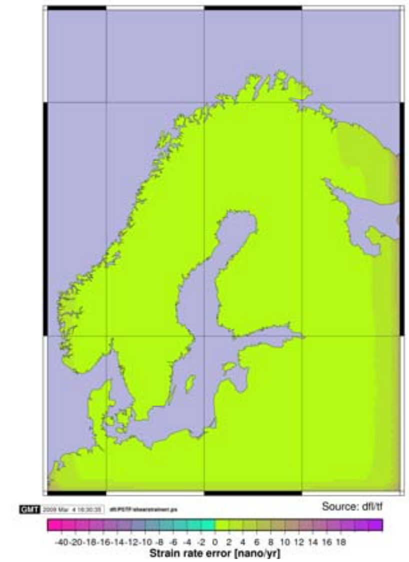
BIFROST project

Shear strain in-plane

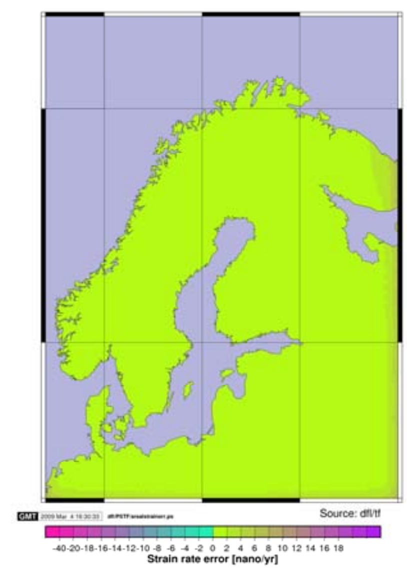
Areal strain in-plane

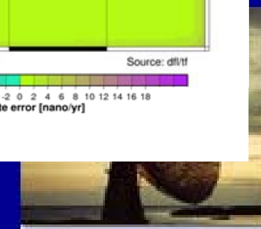


Shear strain



Areal strain

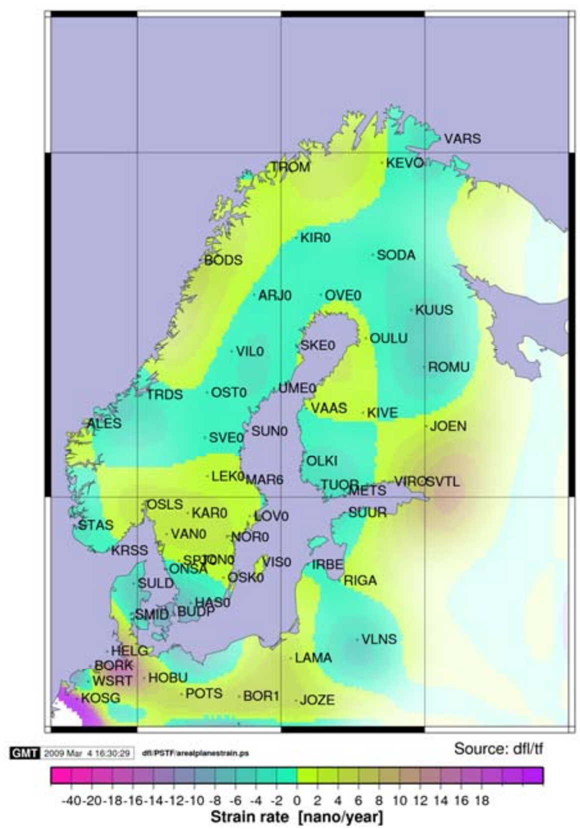




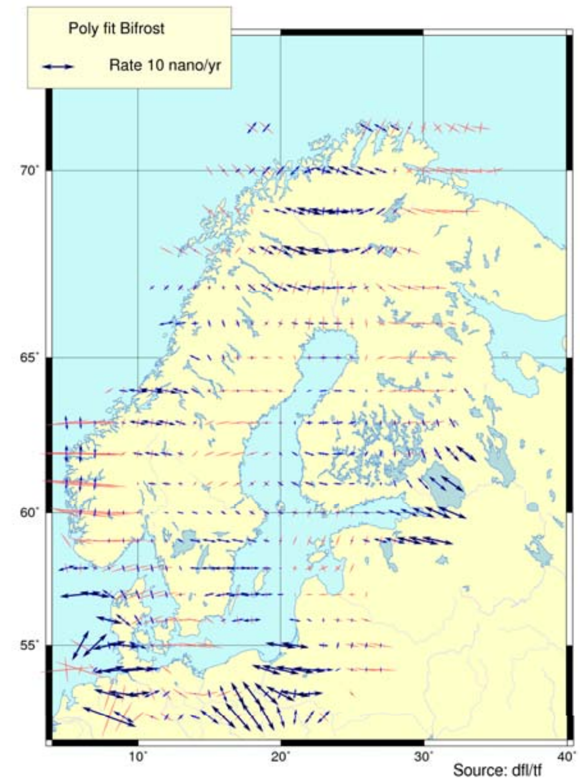
BIFROST project

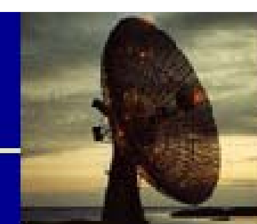
Shear strain in-plane

Areal strain in-plane



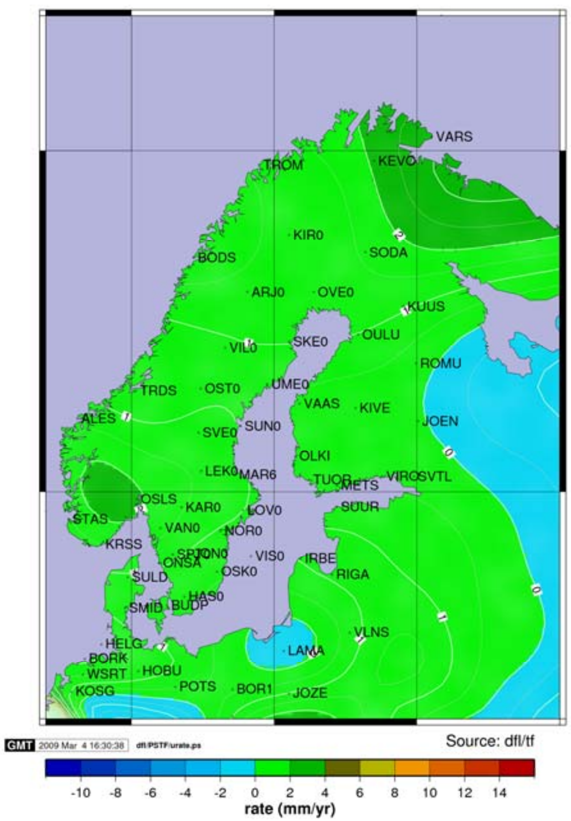
In-plane strain rates



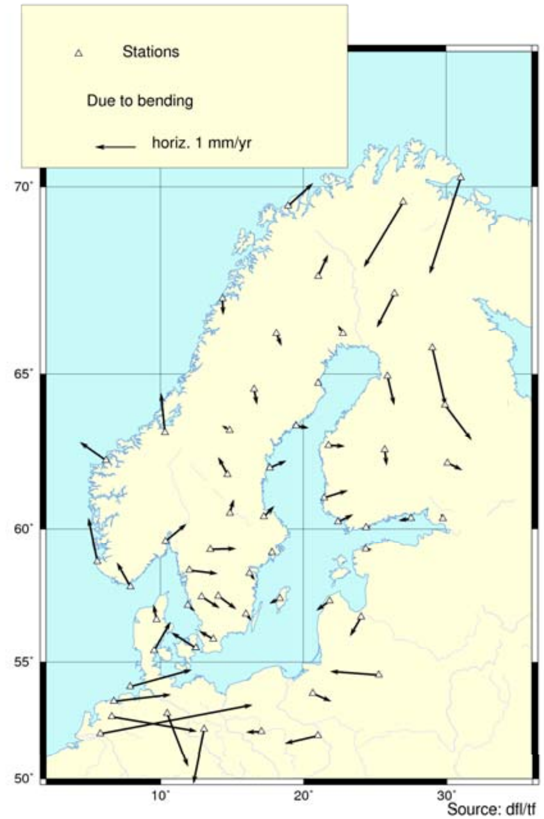


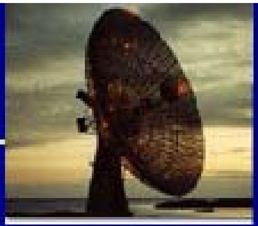
BIFROST project

Polyfit vertical



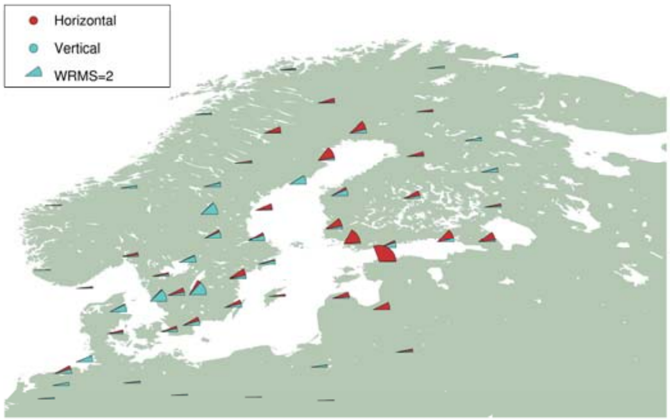
Bending strain rates



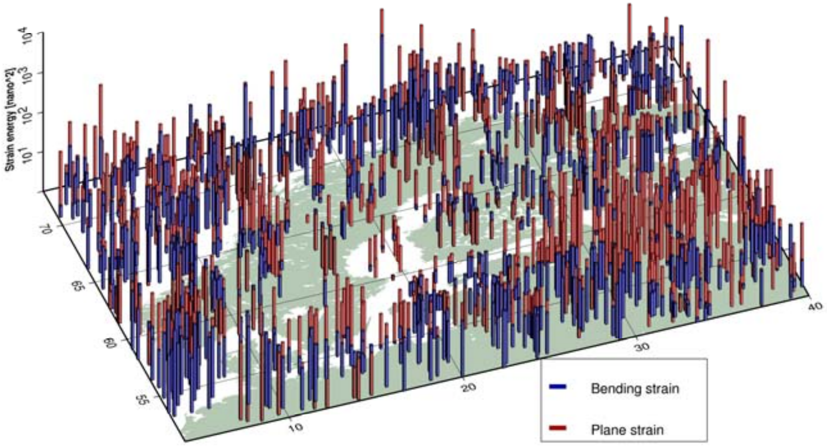


BIFROST project

Displacement model error



Strain Energy



Thick- (thin-) plate model results of observation fit

Input data	Signal wrms	Resid. Wrms
BIFROST-GPS	15.2	0.78 (0.79)
GIA-model	12.6	0.07 (0.04)
GPS minus GIA-model	4.5	0.90 (0.78)

Conclusions

- Early stage of model development
- Clearly, flexure appears important
- Little signal to be explained after subtraction of GIA-model from BIFROST velocities
- Strain rate field looks smooth except for an anomalous area in Finnmarken
- This area is not well sampeled with GPS stations
- In this area also curl is peaking

Deformation energy

Bending: vertical displacement

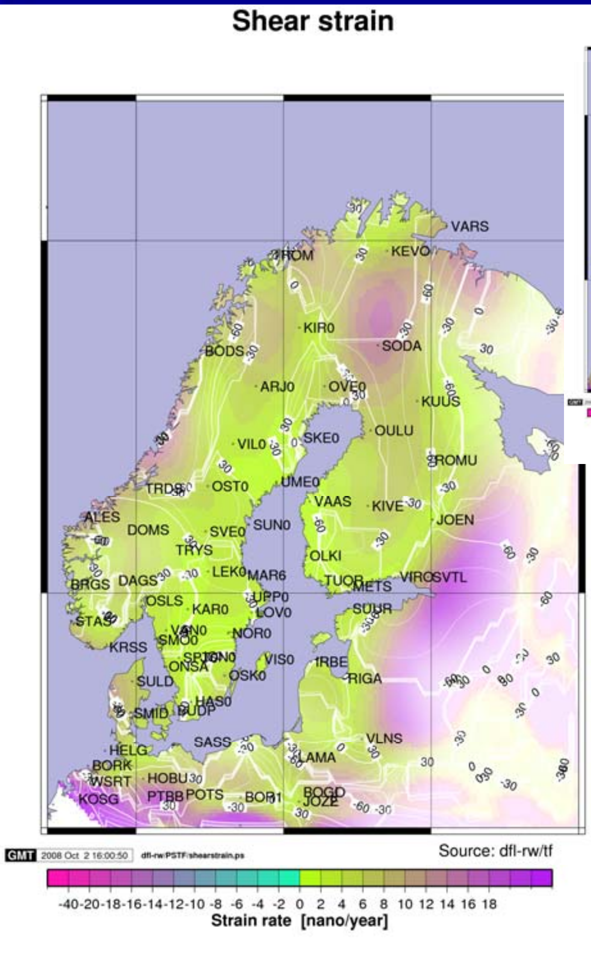
$$E_b = \frac{D^3}{24}\left\{\left[\lambda(1-\nu)^2 + 2\mu(1+\nu^2)\right](\nabla_h^2 u)^2 - 4\frac{\mu(1+\nu)^2}{r^4\sin^2\theta}\frac{\partial^2 u}{\partial\theta^2}\frac{\partial^2 u}{\partial\lambda^2} + 2\frac{\mu}{r^4\sin^2\theta}\left(\frac{\partial^2 u}{\partial\theta\partial\lambda}\right)^2\right\}$$

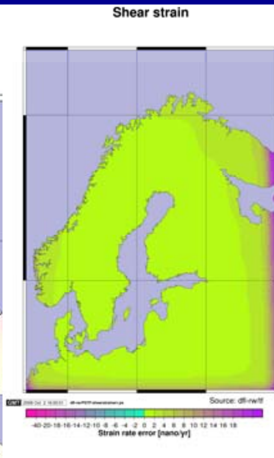
Horizontal deformation: Mid-plane strain

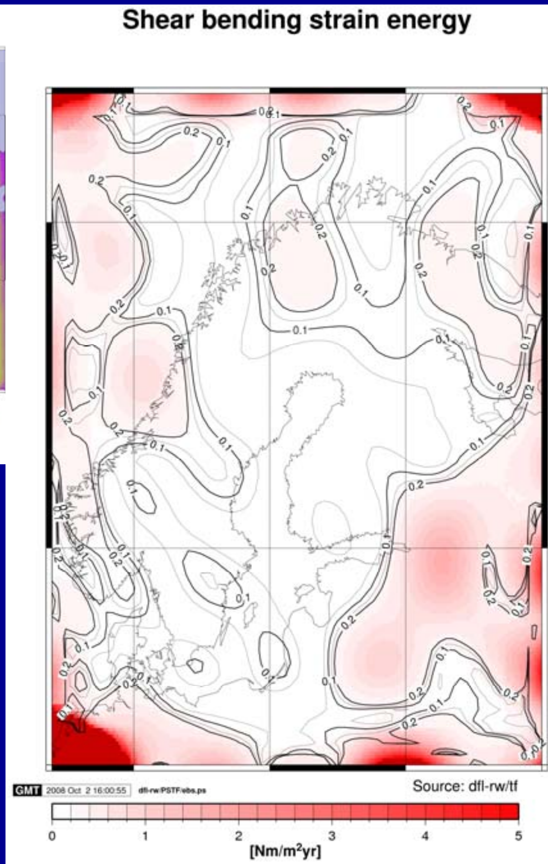
$$E_p = D\left[\frac{\lambda+2\mu}{2}(\epsilon_{\lambda\lambda}^2 + \epsilon_{\theta\theta}^2) + \lambda\epsilon_{\theta\theta}\epsilon_{\lambda\lambda} + \mu\epsilon_{\theta\lambda}^2\right]$$

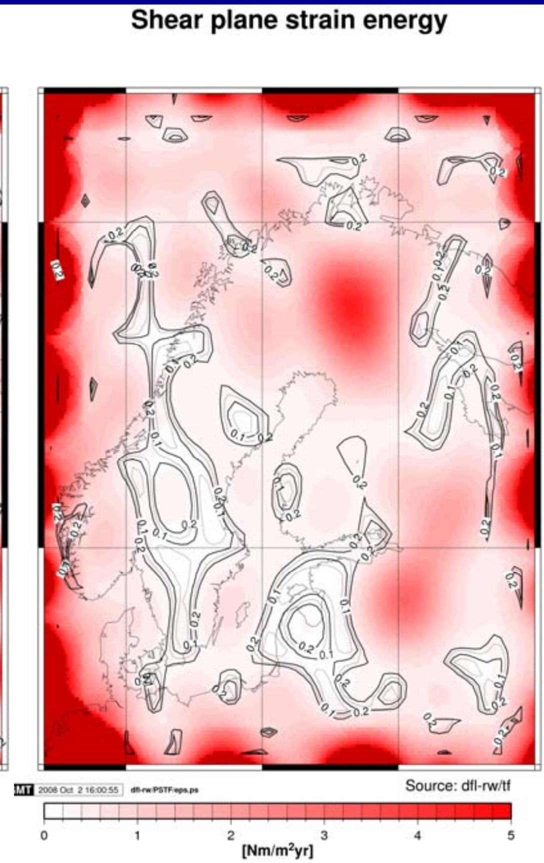
red – compressive; blue: extensive

both: impressive?









BIFROST project